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Soliton solution for Davydov soliton in α-Helix by (G'/G)-expansion technique

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Abstract: In this paper we consider the Davydov soliton in α-Helix proteins having under anti-cubic law of non-linearity and we will solve this non-linear Schrodinger (NLSE) equation with the help of (G'/G)-expansion technique. As special case parabolic law nonlinearity falls out.

Keywords: Non-linear Schr¨odinger equation, Optical solitons, α–Helix, (G'/G)-expansion method.

1. Introduction

In various areas of science, the non-linear Schr¨odinger equation (NLSE) has great significance and studied in different areas of science and numerical solved (Basat & Asghar, 2023). Solitons arise in many areas such as hydrodynamics, solid state physics, biological physics, fiber optic communication, atomic physics and many others (Arnous et al., 2015; Bouzida et al., 2017; Cheemaa et al., 2016; Islam et al., 2017; Liu & Tian, 2012; Mirzazadeh et al., 2015; Rizvi et al., 2016; Tian, 2016, 2017; Younas et al., 2018b; Younis et al., 2016; Younis & Rizvi, 2015) Soliton can propagate over a long distance without altering its shape and attenuation of the amplitude and have balance self-phase modulation through the Group Velocity Dispersion (GVD) (Serge et al., 2017) non-linearity have different forms.

In this article, we focus on the general form of nonlinear media, the under anti-cubic law of non-linearity. Therefore, here in this paper we consider the general form of nonlinearity, which is the under anti-cubic law of nonlinearity.

In this paper we will focus on integrability. There are many forms of integration architectures available in literature. In recent several new methods are developed for finding the soliton solution. Jacobi elliptic function method (Feng et al., 2017), exp-function method (Xu et al., 2016), the modified simple equation method (Feng et al., 2017; Inc & ATEŞ, 2015; İnç et al., 2016; Kilic & Inc, 2015; Tu et al., 2016), tanh-sech method (Inc et al., 2016; Kilic & Inc, 2016; Kilic et al., 2016), extended tanh- method (Tchier et al., 2016, 2017) and so on. In this article we will address another power full and important tool for integration, that is (G'/G)-expansion scheme (Al Qurashi, Ates, et al., 2017; Al Qurashi, Yusuf, et al., 2017; Aslan & Inc, 2017; Aslan, İnç, et al., 2017; Aslan, Tchier, et al., 2017; Inc, Aliyu, & Yusuf, 2017; Inc, Aliyu, Yusuf, et al., 2017; Inc et al., 2016; Kilic & Inc, 2017; Lü et al., 2016; Lü & Lin, 2016; Lü et al., 2018; Tchier et al., 2016).

We will also study the dynamics of soliton in α-Helix proteins. Dynamics of soliton in α-Helix proteins is very important and studied from last few decades (Bouzida et al., 2017; Mirzazadeh et al., 2017; Younas et al., 2018a). From last few decades it gets a lot of progress. In this article we will study the nonlinear Schr¨odinger equation (NLSE) which is referred to as the Davydov module.

2. Analysis for the (G′=G)-expansion method

Here we will discuss the (G'/G)-expansion scheme and then apply on nonlinear Schrödinger equation (NLSE) with under anti-cubic law of nonlinearity. Suppose the form of nonlinear Schrödinger equation (NLSE) is,

$$K(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}) = 0 \quad (2.1)$$

where K represents a polynomial. The (G'/G)-expansion method is given in the following steps:

Step-1:

By using traveling wandering revolution, we suppose that,

$$(x,t) = P(\zeta)\ \zeta = x - ct \quad (2.2)$$

Using Eq.(2.2) in Eq.(2.1) we get,

$$L(P, P', P'', P''' \ldots.) = 0 \quad (2.3)$$

Step-2:

Let suppose that the wandering wave solution of Eq.(3) is in the form of finite series:

$$P(\zeta) = \sum_{q=0}^{n} \alpha_q \left(\frac{G'(\zeta)}{G(\zeta)}\right)^q \quad (2.4)$$

Where α_q are constant with $\alpha_n \neq 0$ and n is positive integer. The function $G(\zeta)$ is the solution of the auxiliary linear ODE.

$$G''(\zeta) + \beta G(\zeta) + \gamma G(\zeta) \quad (2.5)$$

hare β and γ are both real constants.

Step-3:

By balancing the linear term having highest derivative in Eq.(2.3) with the highest order nonlinear term we can find value of n.

Step-4:

Using the general solution of 2nd order ODE given in Eq.(2.5) along Eq.(2.4) in Eq. (2.3). We get an algebraic equation with powers of (G'/G), by setting the coefficients of $(G'/G)^n$ for

$n = 0,1,2,3, \ldots$ to zero. This gives us a system of equations having α_q, β, γ, and c. Then find the values of α_q, β, γ and c from the system of equation. Solution depends on the sign of the discriminant $\Delta = \beta^2 - 4\gamma$. We obtain the solution of Eq.(2.3) and can therefore obtain the exact solutions of equation (2.1).

3. Mathematical Analysis

The governing equation that will be studied for the soliton in α - helix protein is given by.

$$i\Psi_t + \frac{1}{2}\Psi_{xx} + (|\Psi|^{2m} + \Omega|\Psi|^{4m})\Psi = 0 \quad (3.1)$$

In Eq.(3.1), $\Psi(x,t)$ denote the wave profile and the independent variable x is spatial and t is temporal variable and m is perimeter of dual power law nonlinearity. We introduce the traveling wave transformation as,

$$\Psi(x,t) = \rho(\zeta)e^{i[-\kappa x + \omega t + \theta]} \quad (3.2)$$

Where $= x + \kappa t$.

We have

$$\Psi_t(x,t) = [-w\rho(\zeta) + i\kappa\rho'(\zeta)]e^{i[-\kappa x + \omega t + \theta]}$$

$$i\Psi_{xx}(x,t) = [-\kappa^2\rho(\zeta) - 2i\kappa\rho'(\zeta) + \rho''(\zeta)]e^{i[-\kappa x+\omega t+\theta]}$$

$$\Psi|\Psi|^{2m} = \rho^{2m+1}(\zeta)e^{i[-\kappa x+\omega t+\theta]}$$

$$\Psi|\Psi|^{4m} = \rho^{4m+1}(\zeta)e^{i[-\kappa x+\omega t+\theta]}$$

Using Eq.(2.3-2.5) in Eq.(3.1) we get

$$\rho''(\zeta) - (2\omega+\kappa^2)\rho(\zeta) + 2\rho^{2m+1}(\zeta) + 2\Omega\rho^{4m+1}(\zeta) = 0 \tag{3.3}$$

Balancing $\rho''(\zeta)$ with $\rho^{4m+1}(\zeta)$ we get $n+2 = (4m+1)n$ So, $n = \frac{1}{2m}$. Now let suppose that solution of above Eq. (3.3) is of the form:

$$\rho(\zeta) = M\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}}$$

Where M is constant and $G(\zeta)$ satisfies Eq.(2.5) hence we get the following equation.

$$\rho'(\zeta) = -\frac{1}{2m}M\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}+1} - \frac{1}{2m}M\beta\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}} - \frac{1}{2m}M\gamma\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}-1} \tag{3.4}$$

$$\rho''(\zeta) = \left(\frac{1}{4m^2}+\frac{1}{2m}\right)M\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}+2} + \left(\frac{1}{2m^2}+\frac{1}{2m}\right)M\beta\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}+1} + \left(\frac{1}{2m^2}\gamma\ \frac{1}{4m^2}\beta^2\right)M\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}} + \left(\frac{1}{2m^2}-\frac{1}{2m^2}\right)M\beta\gamma\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}-1} + \left(\frac{1}{4m^2}-\frac{1}{2m}\right)M\gamma^2\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}-2} \tag{3.5}$$

Now using previous Equations in Eq.(3.5) and collecting terms which have the same power of $\left(\frac{G'(\zeta)}{G(\zeta)}\right)$ and equal to 0 . So, we get a set of algebraic equation for β,γ,M,ω.

$$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}+2} : \frac{1}{4m^2}\gamma^2M - \frac{1}{2m}\gamma^2M = 0 \tag{3.6}$$

$$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}-1} : \frac{1}{2m^2}\gamma\beta M - \frac{1}{2m}\beta\gamma M = 0 \tag{3.7}$$

$$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}+2} : \frac{1}{4m^2}M + \frac{1}{2m}M + 2\Omega M^{4m+1} = 0 \tag{3.8}$$

$$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}+1} : \quad \frac{1}{2m^2}\beta M + \frac{1}{2m}\beta M + 2M^{2m+1} = 0 \tag{3.9}$$

$$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^{\frac{1}{2m}}:$$

$$\frac{1}{2m^2}\gamma M + \frac{1}{4m^2}\beta^2 M - (2\omega+\kappa^2)M = 0 \tag{3.10}$$

Solving the algebraic equation we get.

$$M = \left(-\frac{(1+2m)}{8\Omega m^2}\right)^{\frac{1}{4m}} \tag{3.11}$$

$$\beta = -\sqrt{\frac{-2m^2(1+2m)}{\Omega(1+m)^2}} \tag{3.12}$$

$$\omega = -\left(\frac{k^2}{2} - \frac{(1+2m)}{4\Omega(1+2m)^2}\right) \tag{3.13}$$

From Eq.(2.5), Eq(3.2), Eq.(3.7) and Eq.(3.13-3.16), we get exact traveling wave solution of the nonlinear schrödingers equation(NLSE) with under anti-cubic law of nonlinearity as:

$$\Psi(x,t) = -\frac{(1+2m)}{2\Omega(1+m)^2}\frac{c_2e^{\sqrt{-2m^2(1+2m)/\Omega(1+m^2)}}}{c_1+c_2e^{\sqrt{-2m^2(1+2m)/\Omega(1+m^2)}}} \times e^{i[-kx+(-k^2/2+(1+2m)/4\Omega(1+m)^2)t+\theta]} \tag{3.14}$$

In Eq.(3.14) if $m = 1$ we get the below exact traveling wave solution:

$$\Psi(x,t) = \left(-\frac{3}{8\Omega}\frac{c_2e^{\sqrt{-6/2\Omega}(x-kt)}}{c_1+c_2e^{\sqrt{-6/2\Omega}(x-kt)}}\right) \times e^{i\left[kx-\left(\frac{k^2}{2}+\frac{3}{16\Omega}\right)t+\theta\right]} \tag{3.15}$$

4 Application of improved (G'/G)-expansion method

Now we are interesting to obtain closed from solution, So by using below transformation:

$$\rho(\zeta) = \chi^{\frac{1}{2m}}(\zeta) \quad (4.1)$$

using eq.(4.1) in eq.(3.5) we get the following ODE

$$2m\chi(\zeta)\chi''(\zeta) + (1-2m)\chi'^2(\zeta) - 4m^2(2\omega + k^2)\chi^2(\zeta) + 8m^2\chi^3(\zeta) + 8m^2\Omega\chi^4(\zeta) \quad (4.2)$$

$$\chi(\zeta) = \sum_{q=1}^{m} \alpha_q \left(\frac{G'(\zeta)}{G(\zeta)}\right)^q \quad (4.3)$$

Here α_q real constants with α_m, m is a positive integer. By balancing the linear term having highest derivative in Eq.(4.2) with the highest order nonlinear term we find value of m, and

$$\chi(\zeta) = \alpha_o + \alpha_1\left(\frac{G'(\zeta)}{G(\zeta)}\right) \quad (4.4)$$

from Eq.(4.4) we have

$$\chi(\zeta) = -\alpha_1\left(\frac{G'(\zeta)}{G(\zeta)}\right)^2 - \alpha_1\beta\left(\frac{G'(\zeta)}{G(\zeta)}\right) - \alpha_1\gamma$$

$$\chi''(\zeta) = 2\alpha_1\left(\frac{G'(\zeta)}{G(\zeta)}\right)^3 + 3\alpha_1\beta\left(\frac{G'(\zeta)}{G(\zeta)}\right)^2 + \alpha_1\beta\mu + \alpha_1\beta^2 + 2\alpha_1\mu\left(\frac{G'(\zeta)}{G(\zeta)}\right)\alpha_1\beta\mu$$

$$\chi^2(\zeta) = \alpha_1^2\left(\frac{G'(\zeta)}{G(\zeta)}\right)^2 + 2\alpha_0\alpha_1\left(\frac{G'(\zeta)}{G(\zeta)}\right) + \alpha_0^2$$

$$\chi^3(\zeta) = \alpha_1^3\left(\frac{G'(\zeta)}{G(\zeta)}\right)^3 + 3\alpha_0\alpha_1^2\left(\frac{G'(\zeta)}{G(\zeta)}\right)^2 + 3\alpha_0^2\alpha_1\left(\frac{G'(\zeta)}{G(\zeta)}\right)^3 + \alpha_0^3$$

$$\chi^4(\zeta) = \alpha_1^4\left(\frac{G'(\zeta)}{G(\zeta)}\right)^4 + 4\alpha_0\alpha_1^3\left(\frac{G'(\zeta)}{G(\zeta)}\right)^3 + 6\alpha_0^2\alpha_1^2\left(\frac{G'(\zeta)}{G(\zeta)}\right)^2 + 4\alpha_0^3\alpha_1\left(\frac{G'(\zeta)}{G(\zeta)}\right) + \alpha_0^4$$

Substituting above set of Equations into Eq. (4.2), and collect all terms which have the same powers of (G'/G) and coefficient of $(G'/G)^i, i = 0,1,2,3,4$ to 0 , we get an algebraic system of equations:

$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^0$:

$$4m\alpha_1^2 + (1-2m)\alpha_1^2 + 8m^2\alpha_1^4\Omega$$

$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^1$:

$$4m\alpha_0\alpha_1 + 6m\alpha_1^2\beta + 2(1-2m)\alpha_1\beta + 8m^2\alpha_1^3 + 32m^2\alpha_0\alpha_1^2\Omega$$

$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^2$:

$$6m\alpha_0\alpha_1\beta + 2m\alpha_1^2\beta^2 + 4m\alpha_1^2\gamma + 2(1-2m)\alpha_1\beta^2 + 2(1-2m)\alpha_1^2\gamma - 4m^2\alpha_1^2(2\omega + k^2) + 24m^2\alpha_0\alpha_1^2 + 48m^2\alpha_0^2\alpha_1^2\Omega$$

$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^3$:

$$2m\alpha_0\alpha_1\beta^2 + 4m\alpha_0\alpha_1\gamma + 2m\alpha_1^2\beta\gamma + 2(1-2m)\alpha_1^2\gamma\beta - 8m^2\alpha_0\alpha_1(2\omega + k^2) + 24m^2\alpha_0^2\alpha_1 + 32m^2\alpha_0^3\alpha_1$$

$\left(\frac{G'(\zeta)}{G(\zeta)}\right)^4$:

$$2m\alpha_0\alpha_1\beta\gamma + (1-2m)\alpha_1^2\gamma^2 - 4m^2\alpha_0^2(2\omega + k^2) + 8m^2\alpha_0^3 + 8m^2\alpha_0^4\Omega$$

solving the above system of equations, we get:

$$\alpha_0 = -\frac{1}{4}\frac{(1+2m)}{(m+1)} + \frac{1}{8}\frac{(\beta\sqrt{-2\Omega(1+2m)})}{m\Omega}$$

$$\alpha_1 = \frac{1}{4}\left(\frac{\sqrt{-2\Omega(1+2m)}}{m\Omega}\right)$$

$$\omega = -\frac{1}{2}\left\{k^2 + \frac{1}{2}\frac{(1+2m)}{\Omega(m+1)^2}\right\}$$

$$\gamma = \frac{1}{4}\left\{\beta^2 + \frac{m^2(1+2m)}{2\Omega(1+m)^2}\right\}$$

β and k are arbitrary constant.

By using above values of ($\alpha_0, \alpha_1, \omega, \gamma$) in Eq.(4.4), solution of Eq.(4.3) takes form:

$$\chi(\zeta) = -\frac{1}{4}\frac{(1+2m)}{(m+1)} \pm \frac{1}{4}\frac{(\beta\sqrt{-2\Omega(1+2m)})}{m\Omega}\left(\frac{\beta}{2} + \frac{G'(\zeta)}{G(\zeta)}\right) \quad (4.5)$$

by putting the general solution of 2nd order linear ODE in Eq.(4.5) we get the following three type of traveling wave solutions.

4.1 Family 1: Trigonometric function solutions

if we have $\Delta = \beta^2 - 4\gamma > 0$ we get the solution in hyperbolic form,

$$\left(\frac{G'}{G}\right) = -\frac{\beta}{2} + \frac{\sqrt{\beta^2-4\gamma}}{2}\left(\frac{A_1\sinh\left(\left(\sqrt{\beta^2-4\gamma}/2\right)\zeta\right) + A_2\cosh\left(\left(\sqrt{\beta^2-4\gamma}/2\right)\zeta\right)}{A_1\cosh\left(\left(\sqrt{\beta^2-4\gamma}/2\right)\zeta\right) + A_2\sinh\left(\left(\sqrt{\beta^2-4\gamma}/2\right)\zeta\right)}\right)$$

By using above values of ($\alpha_0, \alpha_1, \omega, \gamma$), Eq.(4.5), Eq.(4.1) Eq.(3.2) we get

$$\Psi(x,t) = \left[-\frac{1}{8}\frac{(1+2m)}{\Omega(1+m)}\right]2 \mp \left[\frac{A_1\sinh\left(\sqrt{\frac{-m^2(1+2m)}{2\Omega(1+m)^2}}(\zeta)\right) + A_2\cosh\left(\sqrt{\frac{-m^2(1+2m)}{2\Omega(1+m)^2}}(\zeta)\right)}{A_1\cosh\left(\sqrt{\frac{-m^2(1+2m)}{2\Omega(1+m)^2}}(\zeta)\right) + A_2\sinh\left(\sqrt{\frac{-m^2(1+2m)}{2\Omega(1+m)^2}}\right)}\right]^{1/2m} \times e^{i[-kx-(k^2/2+(1-2m)/2\Omega(m+1)^2t+\theta)]}$$

$$\zeta = (x+kt)$$

A_1, A_2 are arbitrary constants. for more results set $A_1 \neq 0$ and $A_2 = 0$ we get solution of the form:

$$\Psi(x,t) = \left[-\frac{1}{8}\frac{(1+2m)}{\Omega(1+m)}\left\{2 \mp \tanh\left(\sqrt{-\frac{m^2(1+2m)}{2\Omega(1+m)^2}}\right)\right\}(x+kt)\right]^{1/2m} \times e^{i[-kx-(k^2/2+(1-2m)/2\Omega(m+1)^2t+\theta)]}$$

Now $A_1 = 0, A_2 \neq 0$

$$\Psi(x,t) = \left[-\frac{1}{8}\frac{(1+2m)}{\Omega(1+m)}\left\{2 \mp \coth\left(\sqrt{-\frac{m^2(1+2m)}{2\Omega(1+m)^2}}\right)\right\}(x+kt)\right]^{1/2m} \times e^{i[-kx-(k^2/2+(1-2m)/2\Omega(m+1)^2t+\theta)]}$$

4.2 Family 2: Trigonometric function solutions

If we have $\Delta = \beta^2 - 4\gamma < 0$ we get the solution in hyperbolic form:

$$\left(\frac{G'}{G}\right) = -\frac{\beta}{2} +$$

$$\frac{\sqrt{4\gamma-\beta^2}}{2}\left(\frac{A_1\sinh\left(\left(\frac{\sqrt{4\gamma-\beta^2}}{2}\right)\zeta\right)+A_2\cosh\left(\left(\frac{\sqrt{4\gamma-\beta^2}}{2}\right)\zeta\right)}{A_1\cosh\left(\left(\frac{\sqrt{4\gamma-\beta^2}}{2}\right)\zeta\right)+A_2\sinh\left(\left(\frac{\sqrt{4\gamma-\beta^2}}{2}\right)\zeta\right)}\right)$$

$$\Psi(x,t) = \left[-\frac{1}{8}\frac{(1+2m)}{\Omega(1+m)}\left[2 \mp i\frac{-A_1\sin\left(\sqrt{\frac{-m^2(1+2m)}{2\Omega(1+m)^2}}(\zeta)\right)+A_2\cos\left(\sqrt{\frac{-m^2(1+2m)}{2\Omega(1+m)^2}}(\zeta)\right)}{A_1\cos\left(\sqrt{\frac{-m^2(1+2m)}{2\Omega(1+m)^2}}(\zeta)\right)+A_2\sin\left(\sqrt{\frac{-m^2(1+2m)}{2\Omega(1+m)^2}}(\zeta)\right)}\right]\right]^{\frac{1}{2m}} \times e^{i[-kx-(k^2/2+(1-2m)/2\Omega(m+1)^2t+\theta)]}$$

Where $\zeta=(x+kt)$ and A_1, A_2 are arbitrary constants. For more results set $A_1 \neq 0$ and $A_2 = 0$ we get solution of the form:

$$\Psi(x,t) = \left[-\frac{1}{8}\frac{(1+2m)}{\Omega(1+m)}\left\{2 \mp i\tanh\left(\sqrt{\frac{m^2(1+2m)}{2\Omega(1+m)^2}}\right)\right\}(x+kt)\right] \times e^{i[-kx-(k^2/2+(1-2m)/2\Omega(m+1)^2t+\theta)]}$$

A_1, A_2 are arbitrary constants.

$$\Psi(x,t) = \left[-\frac{1}{8}\frac{(1+2m)}{\Omega(1+m)}\left\{2 \mp i\coth\left(\sqrt{\frac{m^2(1+2m)}{2\Omega(1+m)^2}}\right)\right\}(x+kt)\right]^{1/2m} \times e^{i[-kx-(k^2/2+(1-2m)/2\Omega(m+1)^2t+\theta)]}$$

4.3 Family 3: Rational function solutions

If we have $\Delta=\beta^2-4\gamma=0$ we get the solution:

$$\left(\frac{G'}{G}\right) = \left(\frac{A_1}{A_1+A_2\zeta}\right)-\frac{\beta}{2}$$

$$\Psi(x,t) = \left[\pm\left(\frac{\sqrt{-2\Omega(1+2m)}}{4m\Omega}\right)\frac{A_1}{A_1+A_2(x+kt)}\right]^{1/2m} \times e^{i[-kx-(k^2/2+(1-2m)/2\Omega(m+1)^2t+\theta)]}$$

The dynamical behavior of this solution has been shown in Figures (1-4) for the following values appeared in Table 1.

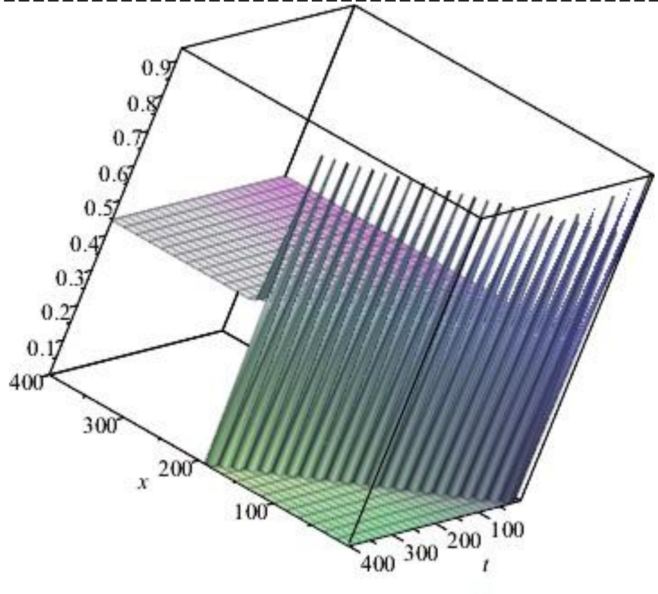

Figure 1: Dynamical solution of $\Psi(x, t)$ at $\Omega = -1$, $n = 1/2$, **$C_1 = 1.5$, $C_2 = -5$.**

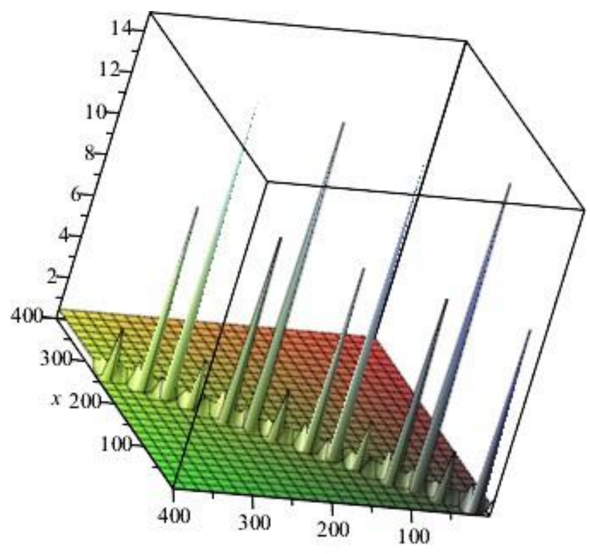

Figure 2: Dynamical solution of $\Psi(x, t)$ at $\Omega = -1$, $n = 1/2$, **$C_1 = -1.5$, $C_2 = 5$.**

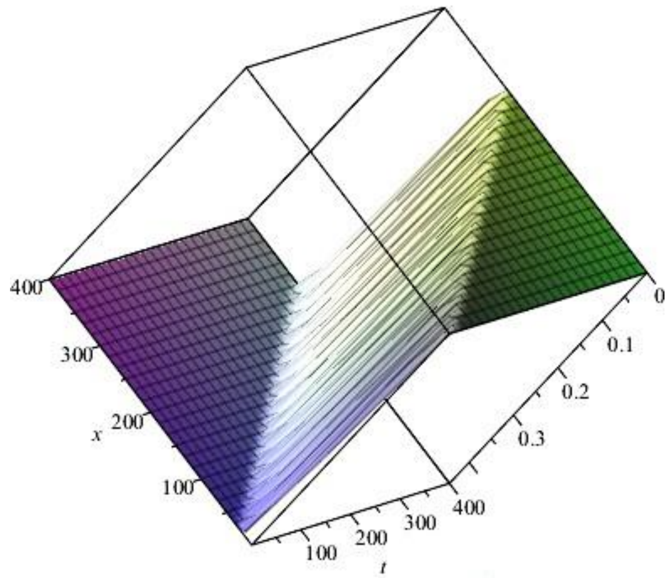

Figure 3: Dynamical solution of $\boldsymbol{\Psi(x,t)}$ at $\boldsymbol{\Omega = -1}$, $n = 1/2$, $\boldsymbol{C_1 = 1.5}$, $\boldsymbol{C_2} = 0.1$.

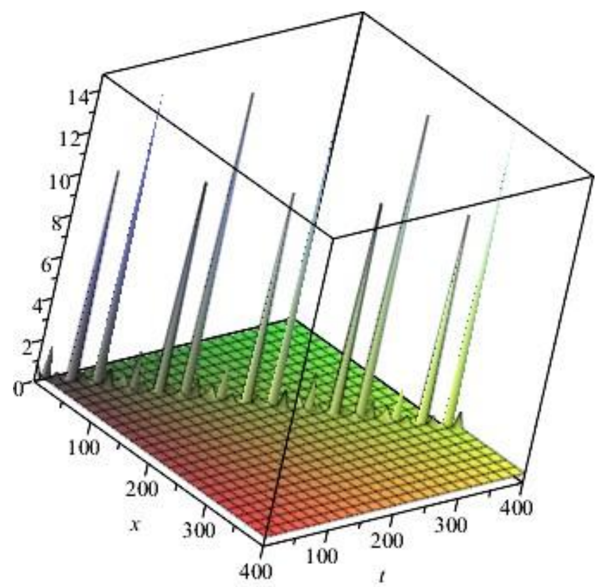

Figure 4: Dynamical solution of $\boldsymbol{\Psi(x,t)}$ at $\boldsymbol{\Omega = -1}$, $n = 1/2$, $\boldsymbol{C_1 = 10}$, $\boldsymbol{C_2} = 5$.

Table 1 Different dynamical values of different solutions

Ω	C_1	C_2	n
-1	1.5	-5	$\frac{1}{2}$
-1	-1.5	5	$\frac{1}{2}$
-1	1.5	0.1	$\frac{1}{2}$
-1	-10	5	$\frac{1}{2}$

Conclusions

In this study, we successfully applied the basic (G′/G) method, which relies on second-order linear differential equations, to derive a series of new and exact soliton solutions for the Davydov model in α-helix proteins. Our findings demonstrate that the (G′/G) method is a powerful and reliable tool for solving complex nonlinear Schrödinger equations, which are fundamental to understanding energy transfer in biological systems. The newly obtained analytical solutions provide a comprehensive theoretical framework and reveal several previously unobserved wave behaviors, offering valuable insights that can be used for both numerical and theoretical studies. Building on this work, future research could apply this method to investigate the effects of external forces on the Davydov soliton. The derived solutions can also be used to explore how quantum effects stabilize these solitons, which could advance our understanding of quantum biology.

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